Nonequilibrium Ionization in Magnetoplasmadynamic Power Generation: Theory and Experiment

Mostafa E. Talaat*
University of Maryland, College Park, Md.

Test results from a closed-loop magnetoplasmadynamic (MPD) generator experiment at gas temperatures in the range of 1000°–1100°F are examined in the light of the theory of magnetoplasmadynamic electrical power generation with nonequilibrium ionization. The effective electrical conductivities measured under virtual short-circuit electrical load conditions in a segmented-electrodes constant-area-duct experiment when the magnetic field is applied, are compared with the thermal equilibrium ionization conductivities, which are calculated using electron densities derived from Saha's equation with the corresponding gas temperatures inserted. This comparison gives a verification of the presence of magnetically-induced nonequilibrium ionization in the MPD generator duct. Next, the energy balance equation is used to derive the electron temperature. The electron temperature is in turn used to calculate the resultant nonequilibrium electron density. This is done using two approaches. One approach uses an ion balance equation, which balances the rate of generation of ions by the hot electrons with the rate of loss of ions by recombination. The other approach derives the electron density from Saha's thermal equilibrium equation using the electron temperature instead of the gas temperature. The calculated nonequilibrium conductivities resulting from the use of these two approaches are compared with those obtained by experiment.

INCREASED world interest in generating more electrical power from nuclear heat has led to the investigation of the concept of closed-loop magnetoplasmadynamic power generation, either as a topper to the steam turbine nuclear power plant (for better over-all power plant efficiency at potentially low capital cost) or as a compact and potentially simple power plant for military, space, or civilian applications in the multimegawatt power range.

An essential prerequisite to the success of the magnetoplasmadynamic power generation principle, in general, is that the working fluid must be rendered adequately electrically conductive. This has been realized in fossil fuel-fired magnetoplasmadynamic generators by seeding the combustion gases with an easily ionized substance (e.g., a potassium compound) and relying on the high temperature of combustion to ionize some of the seed atoms by the thermal equilibrium ionization mechanism. The ionization by this mechanism is accomplished by some of the seed atoms interacting inelastically with some of the few Maxwellian gas species having kinetic energies greater than the ionization potential of the seed atoms. The number of gas species having energies greater than the ionization potential of the seed atoms is, according to the Maxwellian distribution law, exponentially dependant on the gas temperature. The higher the temperature, the greater is the number. The working gas temperatures required for adequate ionization in seeded combustion gases are generally in the 4000°–5000°F range in order to yield electrical conductivities on the order of 10 mho/m at about 1 atm.

Now, an examination of the potential long life, high-temperature (noble), gas-cooled reactor development indicates that the likely magnetoplasmadynamic duct gas temperatures to be attained will be in the 2000°–3000°F range. For example, the Ultra High Temperature Reactor Experiment (UHTREX) at Los Alamos is designed to heat helium up to about 2400°F.[2, 3]

We are thus faced with the problem that, in order to make the magnetoplasmadynamic power generation concept attractive at the temperatures that are feasible for potential gas-cooled nuclear reactors, it is necessary to attain good electrical conductivity in the working fluid at these relatively low temperatures by comparison to combustion temperatures.

The solution to this problem is helped by the fact that in a closed-loop magnetoplasmadynamic system we have more freedom in choosing the working fluid, and we can, e.g., choose one of the high electron mobility noble gases, such as helium or neon, and seed it with the most easily ionizable substance, cesium. Even then, the process of thermal-equilibrium ionization alone would not yield the desirable electrical conductivities that could make magnetoplasmadynamic power generation attractive at the gas temperatures that are compatible with the long time endurance limit of the nuclear fuel materials (viz., 2000°–2600°F).

A nonequilibrium ionization process similar to that which is taking place in fluorescent light tubes could lead us to useful electrical conductivities in seeded noble gases at gas temperatures compatible with a long-life gas-cooled reactor.

In general, the approach of nonequilibrium ionization as applied to magnetoplasmadynamics involves injecting into the working fluid the necessary power to generate and maintain a positive ion-electron pair density higher than that produced by thermal equilibrium ionization alone. When the ionizing power is turned off, the ion-electron pair density subsides to the level attained with thermal equilibrium ionization alone.

One can use several methods of injecting nonequilibrium ionizing power, e.g., the d.c. or a.c. discharges or the injection into the working fluid of high energy electrons, particles, or photons of the proper frequency band. Most of these methods would involve special external power supplies and special design provisions to introduce the ionizing power into the working medium.

A simple, and hence, attractive method would accomplish the feeding of the ionizing power into the magnetoplasmadynamic generator working medium by the mere flow of the load current electrons through the plasma under the influence of the electric field, which is induced by the interaction of the magnetic field with the moving conductive medium. Or, in other words, letting the internal Joule heat generated in the working medium by the current flow provide the ionizing power to maintain the nonequilibrium electrical

Presented as Preprint 64-379 at the 1st AIAA Annual Meeting, Washington, D. C., June 29–July 2, 1964; revision received January 29, 1965.

* Professor of Mechanical Engineering.

conductivity at the given load current. This can be the case if the electron-gas species collision frequency is made sufficiently low so as to necessitate that the electrons assume a much higher average energy than that of the gas species in order to be able to transmit to the gas molecules the internal Joule heat generated per second. The electrons thus must maintain themselves at a higher average energy than that represented by the bulk gas temperature. The result is that more electrons will now have a greater probability of ionizing the seed atoms, since a greater number of them, according to the statistical distribution law, will have energies greater than the ionization potential of the seed atoms, and more electron pairs will thus be generated. Hence, a higher electrical conductivity will be attained than is possible with the thermal equilibrium ionization process alone. This mechanism of ionization is generally called magnetically induced nonequilibrium ionization, since it refers to the ionization process resulting from the electric field, which is magnetically induced. Its attractiveness is in its simplicity. Its experimental demonstration under actual operating conditions became one of the primary objectives of closed-loop magnetoplasmadynamic research programs directed toward the realization of magnetoplasmadynamic power plants at gas temperatures compatible with the long time endurance limit of heat source and containment materials.

This paper presents and discusses the data obtained during runs made in December 1963 at the closed-loop research magnetoplasmadynamic facility at the Martin Company, Baltimore, Md. Their scientific value lies in the fact that they give an early experimental indication of the presence of magnetically induced nonequilibrium ionization. These runs were made with helium seeded with cesium under closed-cycle magnetoplasmadynamic generator operating conditions. The gas temperatures during these runs were varied from about 800°–870°K or 1000°–1100°F, and in each case the effective electrical conductivities measured whenever the magnetic field was on, although only on the order of a mmho/m, were over two orders of magnitude higher than the gas conductivities that could possibly have been obtained by thermal equilibrium ionization alone, being on the order of a μmho/meter. The results were reproduced within about 10% whenever the same magnetic field intensity was applied: seven times for one value of magnetic field intensity, four times for another value, and two times for a third value, under substantially the same gas temperature and pressure conditions. Whenever the direction of the magnetic field was reversed, the direction of the load current between each two opposite electrodes also was reversed, indicating the dependance of the measured current on the magnetic field and that the observed effect was magnetically induced.

Brief Description of the Magnetoplasmadynamic Closed-Loop Research Facility

The experimental closed-loop magnetoplasmadynamic power generation research facility used in obtaining these results was built at the Martin Company, Baltimore, Md. (for a detailed description, see Ref. 11). It included a helium compressor, which moved the gas through the system. The gas was heated to over 800°K before it was passed over a getter to reduce its impurities to a minimum, whereupon it was seeded with cesium. The mixture of helium and cesium was heated further as it passed through the main heater. The hot mixture was then expanded as it passed through a nozzle and part of its thermal energy was converted into kinetic energy at Mach number 0.3–0.56. The fast-moving, hot mixture then flowed through the alumina-lined electrical power generation duct where its level of ionization was raised above that of thermal equilibrium, and where the electrical power was generated by interaction of the moving, ionized cesium-seeded helium gas mixture with the cross magnetic field produced by an electromagnet.

The power generation duct had seven pairs of segmented iridium electrodes, which permitted operation as a segmented electrode Faraday or Hall generator. The dimension of the duct was 0.635 cm × 5.08 cm × 12.4 cm, and each electrode had an area of 0.605 cm². The electromagnet was capable of generating magnetic field intensities of up to about 29 kgauss.

Comparison Between Theory and Experiment

When the assumptions are made that the gas conductivity is uniform, that the electron density is uniform, that the effect of ion slip is negligible, and that the net charge density is also negligible, then the generalized Ohm's law leads, for the case of segmented-electrode magnetoplasmadynamic generator, where the current flow is permitted only in the direction of flow (x direction) or in the direction between opposite electrode pairs (y direction), to the following familiar expressions for the x and y components of current densities, respectively:

$$J_x = \sigma_0(1 + \beta^2)^{-1}[\epsilon_x + \beta(uB - \epsilon_y)] \quad (1)$$

$$J_y = \sigma_0(1 + \beta^2)^{-1}[\beta\epsilon_x - (uB - \epsilon_y)] \quad (2)$$

where J_x and J_y are the x and y components of current densities, respectively; σ_0 is the scalar conductivity of the gas; ϵ_x and ϵ_y are the electric fields in the x and y direction, respectively; u is the gas velocity in the x direction; B is the applied magnetic field in the z direction; and $\beta = \mu_e B$ where μ_e is the mobility of the electrons.

Under the ideal conditions of the infinitely segmented electrodes, the current density in the x direction (J_x) can be made to vanish if there is no electric load connection between the upstream and downstream electrode pairs (viz., a connection for a Hall generator mode of operation in which $I_H = 0$). We can see from Eq. (1) that, when the condition of $J_x = 0$ is satisfied, the electric field in the x direction will approach the value ϵ_{x0} as given by

$$\epsilon_{x0} = -\beta(uB - \epsilon_y) \quad (3)$$

Substitution of Eq. (3) into Eq. (2) then gives us the value that the current density in the y direction (viz., between opposite electrode pairs) approaches as the ideal condition $J_x = 0$ is satisfied. Thus,

$$J_{y0} = -\sigma_0(uB - \epsilon_y) \quad (4)$$

When the ratio of the length of an electrode segment L_E, parallel to the direction of gas flow (viz., the x direction), to the separation D, between the opposite electrode segments (parallel to the y direction) is appreciable, viz., for the case of finitely segmented electrodes, the current density in the x direction does not vanish even when there is no external Hall generator load current (i.e., even when $I_H = 0$). The effect of finite electrode segmentation may thus be considered as if the electrodes were still infinitesimally divided, but with a finite resistance between each other such that a current density in the x direction $J_x \neq 0$ exists within the generator, even when there is no external load current in the Hall mode of operation (viz., even when $I_H = 0$). In other words, the finitely segmented electrodes may be considered as behaving between the case of perfectly solid and the case of ideally infinitesimally segmented electrodes. The effective electrical field in the x direction may now be expressed in terms of a reduction factor $0 \leqslant r \leqslant 1$ (with $r = 0$ for solid electrodes and $r = 1$ for infinitely segmented electrodes when $J_x = 0$), as follows:

$$\epsilon_{xe} = -r\beta(uB - \epsilon_y) \quad (5)$$

The expression for the effective current densities in the x and y directions which would be consistent with the previous discussions and Eq. (5) may now be written, respectively, as follows:

$$J_{xe} = \sigma_0(1 + \beta^2)^{-1}[\epsilon_{xe} + \beta(uB - \epsilon_y)] \tag{6}$$

$$J_{ye} = \sigma_0(1 + \beta^2)^{-1}[\beta\epsilon_{xe} - (uB - \epsilon_y)] \tag{7}$$

Furthermore, if we define an effective Hall generator mode of operation load factor in the x direction by the ratio

$$e_H = \epsilon_{xe}/r\beta(uB - \epsilon_y) \tag{8}$$

note that this load factor may still be smaller than 1 even under open circuit Hall generator mode of operation if there are end leakage losses due to a leakage path between the outer electrodes and the adjacent earthed portion of the magnetoplasmadynamic duct system. For example, if s_1 and s_2 are the distances between the upstream and downstream electrodes and the adjacent earthed portions of the magnetoplasmadynamic duct, respectively, and σ_{s1} and σ_{s2} are the corresponding conductivities of the gas in these regions, then the Hall load factor due to end leakage, even under zero external load, may be given by the following expression, which is derived using a similar treatment to that given in Ref. 4:

$$\frac{1}{e_{H0}} = 1 + \left(\frac{\sigma_{s1}}{\sigma_0}\right)\left(\frac{L}{2s_1}\right)(1 + \beta^2)\left[\frac{1 + (s_1/s_2)}{1 + (\sigma_{s1}/\sigma_{s2})}\right] \tag{9}$$

By also defining a load factor e_L in the y direction by

$$e_L = (\epsilon_y/uB) \tag{10}$$

and using Eqs. (8) and (10) in Eqs. (6) and (7), the expression for the equivalent current densities in the x direction and y directions may be written in terms of r, e_H, and e_L as in the following forms, respectively:

$$J_{xe} = \beta\sigma_0(1 + \beta^2)^{-1}(1 - re_H)uB(1 - e_L) \tag{11}$$

$$J_{ye} = -(1 + \beta^2 re_H)\sigma_0(1 + \beta^2)^{-1}uB(1 - e_L) \tag{12}$$

and since $\sigma_0 = en_e\mu_e$ and $\beta = \mu_e B$, (11) and (12) may be alternately written as follows (after putting $re_H = \theta$):

$$J_{xe} = \beta^2(1 + \beta^2)^{-1}(1 - \theta)(1 - e_L)(enu) \tag{13}$$

$$J_{ye} = -\beta(1 + \beta^2)^{-1}(1 + \beta^2\theta)(1 - e_L)(enu) \tag{14}$$

From Eqs. (13) and (14), the ratio $J_{xe}/(-J_{ye})$ for any electrode pair along the duct length is now given by

$$(J_{xe})/(-J_{ye}) = \beta(1 - \theta)/(1 + \beta^2\theta) \tag{15}$$

We can see from Eq. (15) that this ratio is a function of $\beta = \mu_e B$ and the over-all effective Hall generator load factor θ with respect to the given electrode pair position under consideration which is itself also a function of β and geometry. Equation (15) may also be used to determine this over-all effective Hall load factor θ, once J_{xe}, J_{ye}, and B are measured and μ_e is estimated for the working medium used at the gas conditions at the electrode pair position under consideration. Note that, for a true short-circuited Hall generator, e_H would be zero, and the ratio $(J_{xe})/(-J_{ye}) = \beta$. However, a residual Hall field may exist with respect to a given electrode pair along the duct, because of the variation in electron density along the duct even when an external short circuit is applied between the outermost upstream and the downstream electrode pairs. This would make the ratio between the effective current densities differ considerably from the factor β.

Let us now apply the preceding discussion to the following set of test conditions for a selected electrode pair (electrode pair #7):

1) The current density J_{ye}, measured between the opposite electrode pair #7 under a virtual external short-circuit connection between them (viz., $e_L = 0$), is

$$J_{ye} = \frac{I_7}{2L_E G} = \frac{1.47 \times 10^{-4}}{1.21 \times 10^{-4}} = 1.215 \text{ amp/m}^2 \tag{16}$$

2) The corresponding current density J_{xe}, measured between the externally virtually short-circuited electrode pairs #1 and #7, is

$$J_{xe} = I_H/GD = 1.05 \times 10^{-4}/3.22 \times 10^{-4} = 0.326 \text{ amp/m}^2$$

The ratio $(J_{xe}/-J_{ye}) = (0.326/1.215) = 0.268$ or $(1/3.73)$. Thus, from Eq. (15), we have

$$3.73 = (1 + \beta^2\theta)/(1 - \theta)\beta \tag{17}$$

Now, if we can estimate β, we can determine the equivalent residual Hall load factor θ with respect to the conditions at the electrode pair #7. Now the corresponding gas temperature and pressure measured near electrode pair #7 were $T = 863°\text{K}$ and $p_g = 0.353 \times 10^5$ newtons/m². (For lower temperature tests see Table 1.) The helium mass flow rate $\dot{m}_g = 6$ gm/sec, and the cesium seed injection rate $\dot{m}_s = 0.23$ gm/sec $= 0.115$ mole-%. The helium gas density $\rho_g = 19.65$ gm/m³ corresponding to a helium atom density $n_g = 2.97 \times 10^{24}$ atoms/m³ and a cesium seed atom density $n_s = 3.42 \times 10^{21}$ atoms/m³. The gas velocity $u_7 = 946$ m/sec, the Mach number $M_7 = 0.558$, and the magnetic field intensity $B = 1.05$ webers/m².

Since the ion density n_p was only on the order of 10^{16}/m³, the scattering effect of the ions on the mobility may be neglected by comparison to that due to the gas and seed neutrals, and the electron mobility μ_e in helium seeded with cesium may be calculated using formulas given in Ref. 1, when the effect of the ions is neglected, viz.,

$$\mu_e = (e/m_e\nu_c) \tag{18}$$

where

$$\nu_c = v_0(1.33\, q_g n_g + q_{0s} n_s) \tag{18a}$$

where $v_0 = 0.593 \times 10^6\, (E_e)^{0.5}$ m/sec (E_e being the electron temperature in volts), q_g is the electron helium atom collision cross section assumed to be a constant $= 5.5 \times 10^{-20}$ m², and q_{0s} is the electron cesium atom cross section assumed to be a function $= 230 \times 10^{-20}/(E_e)^{0.5}$ m². Substituting the preceding numerical values, the collision frequency is

$$\nu_e = 1.278 \times 10^{11}[(E_e)^{0.5} + 0.0362] \text{ sec}^{-1} \tag{19}$$

and the mobility is

$$\mu_e = 1.37/[0.0362 + (E_e)^{0.5}] = 1.37/(0.036 + 0.273a) \tag{19a}$$

where $a^2 = (E_e/E_g) =$ ratio of electron temperature E_e to the gas temperature $E_g = 0.0745$ v. If we estimate (E_e/E_g) to be approximately equal to 1.6 (an estimate that will be checked later when we calculate E_e; note at this time, however, that μ_e is not too sensitive to reasonable departure in the value of (E_e/E_g)), then $\mu_e = 3.59$ m²/weber, and $\beta = 1.05 \times 3.59 = 3.77$. Now by solving Eq. (17), we get for the effective Hall load factor a value of 0.462.

The electron temperature E_e may now be calculated by equating the Joule heat in the gas $(1/\sigma_0)(J_{xe}^2 + J_{ye}^2)$ with the electron elastic and inelastic collision losses. If we neglect all collision losses except those elastic collision losses with the helium neutrals, then from Ref. 1

$$(1/\sigma_0)(J_{xe}^2 + J_{ye}^2) = \nu_{cg}(96/9\pi)(m_e/m_g)(E_e - E_g)en_e \tag{20}$$

where

$$\nu_{cg} = v_0(1.33\, q_g n_g) \tag{20a}$$

for a gas with constant electron collision cross section.

Using Eqs. (11) and (12) for J_{xe} and J_{ye} in Eq. (20) and rearranging the terms, we get the following equation for the electron temperature E_e:

Table 1 Reproducibility of data at the magnetic field intensity of B = 0.52 weber/m²

Measured data	Measurement number 1	2	3	4
Helium mass flow rate $\dot{m}_g$, g/sec	6.25	6.25	6.25	6.25
[a] Load current between one electrode pair (#7) I_E, μA	−24	−28	−31.5	−32.5
Electrode temperature T_E, °K (thermocouple near electrode #7)	780	770	805	805
Gas temperature T_g, °K	812	802	840	840
Gas pressure p_g, 10^5 newtons/m² (pressure probe near electrode #7)	0.398	0.375	0.332	0.332
Gas atom density n_g, 10^{24} atoms/m³	3.55	3.39	2.86	2.86
Gas velocity u, m/sec	820	860	1020	1020
Magnetically induced electric field uB, v/m	426	447	530	530
[b] Measured effective electrical conductivity, $\sigma_0 = (I_E/2A_EuB)\ 10^{-6}$ mho/m	466	517	491	507
[c] Calculated thermal equilibrium ionization conductivity at T_g, σ_g, 10^{-6} mho/m	0.321	0.184	0.725	0.725
Ratio (σ_0/σ_g) (the magnetically induced ionization effect)	1450	2810	667	700

[a] The reversal of the current sign corresponds to the reversal of the magnetic field.
[b] Based on an area for the flow of current between each electrode pair, corresponding to the total pitch between electrodes (this area is twice the actual electrode area and thus leads to a conservative value of the conductivity).
[c] The electron density n_e is calculated from Saha's equation for thermal equilibrium using T_g, and the conductivity is calculated using Eq. (23), Chap. III of the First Semi-Annual Report, MND-2939 (see Ref. 11, also Ref. 1).

$$E_e = E_g[1 + (\gamma 3\pi/32)\beta^2(1 + \beta^2\theta^2) \times (1 + \beta^2)^{-1}M^2(1 - e_L)^2] \quad (21)$$

The ratio (E_e/E_g) estimated previously may now be checked by substituting the values of β and θ used before in Eq. (21). Thus $(E_e/E_g) = 1.578$. Now by reiterating, using $(E_e/E_g) = 1.58$, we get $\mu_e = 3.615$ m²/weber, $\beta = 3.80$, and $\theta = 0.461$. Using these values again in Eq. (21), we get $E_e/E_g = 1.58$ or $E_e = 1.58$, $E_g = 0.118$ v.

We can now solve Eq. (12) to obtain the gas conductivity σ_0, yielding

$$\sigma_0 = -J_{ye}(1 + \beta^2)/(1 + \beta^2\theta)uB = 2.47 \times 10^{-3} \text{ mho/m}$$

The electron density may also be calculated from the relation

$$n_e = \sigma_0/e\mu_e = 0.427 \times 10^{16} \text{ electron/m}^3$$

At a higher magnetic field $B = 2.1$ webers/m², the following operating conditions existed:

$$J_x = 0.1924 \text{ amp/m}^2 \qquad J_y = 0.504 \text{ amp/m}^2$$

$$(J_y/J_x) = 2.62$$

$$p_g = 0.338 \times 10^5 \text{ newtons/m}^2 \qquad T = 853°\text{K}$$

$$\rho = 0.0197 \text{ kg/m}^3$$

$$\dot{m}_g = 6 \text{ gm/sec} \qquad u = 944 \text{ m/sec} \qquad M = 0.558$$

$$n_g = 2.88 \times 10^{24} \text{ atoms/m}^3$$

$$n_s = 3.31 \times 10^{21} \text{ atoms/m}^3$$

and if we take $E_e/E_g = 1.67$, we get $\mu_e = 3.67$ m²/weber. Since $\beta = 7.7$,

$$\theta = \{(J_y/J_x)\beta - 1\}/\beta\{(J_y/J_x) + \beta\} = 0.241$$

Now we check (E_e/E_g) using Eq. (21), thus $(E_e/E_g) = 1.669$ or $E_e = 0.123$ v. Again, from Eq. (12), if $e_L = 0$, the gas conductivity corresponding to these operating conditions is $\sigma_0 = (-J_{ye})(1 + \beta^2)/uB(1 + \beta^2\theta) = 10^{-3}$ mho/m, and the corresponding electron density is $n_e = 0.17 \times 10^{16}$ electrons/m³.

Note that the electron density and conductivity for this case ($B = 2.1$ webers/m²), although they are of the same order of magnitude as when $B = 1.05$ webers/m², are numerically less although the electron temperature E_e at $B = 2.1$ webers/m² is slightly greater (0.123 v) than that at $B = 1.05$ weber/m² (0.118 v). This slight inconsistency may be explained as due to the likelihood that a small residual electric field may have existed at the higher magnetic field value, which made $e_L \neq 0$ but say $= 0.1$. If this were the case, then Eq. (21) gives $(E_e/E_g) = 1.542$, or $E_e = 0.114$ v, which is less than E_e at $B = 1.05$ webers/m².

Let us next compare the electron density measured with that calculated from theoretical considerations using the derived electron temperature $E_e = 0.118$ v. We will carry out the calculations first by substituting $E_e = 0.118$ v into Saha's thermal equilibrium equation and second by balancing the rate of ion generation, due to resonant radiative excitations and collisional ionization from the excited states, with the rate of recombination. The results will be compared and discussed.[1]

Substitution of $E_e = 0.118$ v into Saha's equation [Eq. (23) of Ref. 1], we get $K_{ne} = 3 \times 10^{27} E_e^{1.5} \exp(-V_i/E_e) = 0.571 \times 10^{12}$ m^{-3}; hence, $n_e \approx (n_{s0}K_n)^{0.5} = 4.42 \times 10^{16}$ electrons/m³, which is about an order of magnitude higher than the value of 0.427×10^{16} electrons/m³, which is derived from the test results.

The thermal equilibrium electron density n_{e0} may be calculated from Saha's equation using the measured gas temperature $E_g = 0.0745$ v. Thus $K_{ng} = 3 \times 10^{27} E_g^{1.5} \exp(-V_i/E_g) = 0.618 \times 10^4$ per m³; hence $n_{e0} = (K_{ng}n_{s0})^{0.5} = 0.46 \times 10^{13}$ electrons/m³, which is seen to be below the value of $n_e = 0.427 \times 10^{16}$ electrons/m³, which is derived from the test results, by more than two orders of magnitude.

In Ref. 1, the single step collisional ionization coefficient from the ground state s_i is given by

$$s_i = 0.67 \times 10^6(E_e)^{0.5}(k\beta_i) \times (V_i + 2E_e)\exp(-V_i/E_e)\text{m}^3/\text{sec} \quad (22)$$

where $(k\beta_i)$ is the collisional ionization cross section in m²/v for electron energies greater than eV_i. Values of $(k\beta_i)$ for cesium appearing in the literature[5,6] fall in the range of 10^{-20}–10^{-19} m²/v. If we take an effective $(k\beta_i) = 1.0 \times 10^{-19}$ m²/v, $E_e = 0.118$ v, and $V_i = 3.89$ v, we get $s_i = 4.45 \times 10^{-28}$ m³/sec.

Since the electron density is low, it is likely that the recombination coefficient is primarily due to the radiative capture. A formula of recombination coefficient for cesium based on the radiative capture cross section to the 6p level, as measured by Mohler, was given in Ref. 1, Eq. (32) [$\alpha_{r-6p} = 2.02 \times 10^{-20}/(E_e)^{0.5}$]. Assuming that the contribution from capture to the ground state and other states would raise the value of the radiative recombination coefficient by at most an order of magnitude (see, e.g., Ref. 7), we get, for an electron temperature of 0.118 v, a radiative recombination coefficient ($\alpha_r = 2.02 \times 10^{-19}/(E_e)^{0.5}$m³/sec, or† $\alpha_r = 0.558$

† Compare this value with the value given by Dewan[8] for hydrogenlike atoms, viz., $\alpha = 6.7 \times 10^{-19}/(E_e)^{0.5}$ which yields an $\alpha_r = 1.95 \times 10^{-18}$ m³/sec at $E_e = 0.118$.

$\times 10^{-18}$ m³/sec. The value of $[s_i n_{s0}/(s_i + \alpha_r)]$ is now calculated to be 0.269×10^{13} and is less than the thermal equilibrium electron density, indicating that the contribution to the electron density by collisional ionization from the ground state of the cesium seed atoms is small.[1]

Let us next examine the level of ionization resulting from electron collisions with the population of excited atoms. These excited atoms are in turn generated by capture of radiation power from the duct walls over their respective resonance bandwidth. Now, the intensity of radiation from ideal blackbody is given by

$$I_\lambda \approx (h\nu^5/c^3) \exp-(h\nu/kT) \text{ w/m}^3 \qquad h\nu \gg kT \tag{23}$$

where h is Planck's constant, ν is the radiation frequency, and c is the speed of light. The radiative power dW in w/m², which is emitted from an ideal blackbody over the solid angle 2π, is therefore given by

$$dW = 2\pi I d\lambda \tag{24}$$

If the resonance absorption bandwidth of the seed atoms for a resonance excitation level is $\Delta\lambda_0$ and the spectral emissivity of the duct wall is ϵ_λ, then the power absorbed per unit volume over the resonance bandwidth $\Delta\lambda_0 = \lambda_2 - \lambda_1$, is given by

$$W_{\text{abs}} = \int_{\lambda_1}^{\lambda_2} \frac{2(G+D)}{GD} 2\pi\epsilon_\lambda I_\lambda d\lambda = \frac{2(G+D)}{GD} 2\pi\epsilon_\lambda eV^* \left(\frac{c}{\lambda_0^3}\right)\left(\frac{\Delta\lambda_0}{\lambda_0}\right) \exp\left(\frac{-V_0}{E_g}\right) \frac{\text{w}}{\text{m}^3} \tag{24a}$$

(since all the power emitted from the walls over the bandwidth $\Delta\lambda_0$ is absorbed), and

$$(\nu/c) = (1/\lambda) \qquad (\lambda_1 + \lambda_2)/2 = \lambda_0 \qquad eV_* = h\nu_0$$

If the population of the excited states of resonance wave length λ_0 is n_* and their average resonance imprisonment lifetime is τ_*, then the power radiated by these excited states is given by

$$W_{\text{rad}} = (n_*/\tau_*)eV_* \tag{25}$$

Since, under steady-state, the power absorbed [Eq. (24a)] is balanced by the power radiated [Eq. (25)] by the population of excited states, viz., $W_{\text{abs}} = W_{\text{rad}}$, the density $n*$ of these excited states is therefore given by

$$(n_*/n_s) = (2\pi\epsilon_\lambda/n_s)(c\tau_*/\lambda_0^3) \times (\Delta\lambda_0/\lambda_0) \exp(-V_*/E_T)[2(G+D)/GD] \tag{26}$$

Thus if we consider that the dominant resonance levels are the $6p$ levels, viz., the resonance level with $\lambda_0 = 8521$ Å ($V_* = 1.45$ v) and the resonance level with $\lambda_0 = 8943$ Å ($V_* = 1.38$ v),[9] and using the values $\Delta\lambda_0 = 50$ Å, $\tau_* = 10^{-3}$ sec, $\epsilon_\lambda = 0.45$, $q_\lambda = 6.5 \times 10^{-19}$ m² (obtained from the results reported in Ref. 9) for the cesium atom density n_{s0} of the experiment, viz., 3.42×10^{21} atoms/m³, we get for n_{1*} ($\lambda_0 = 8521$ amp, $V_{1*} = 1.45$ v, and $E_T = 0.0745$ v) ($n_{1*}/n_s = 0.295 \times 10^{-5}$) and likewise for n_{2*} ($\lambda_0 = 8943$ Å, $V_{2*} = 1.38$ v, and $E_T = 0.0745$ v) $n_{2*}/n_s = 0.625 \times 10^{-5}$.

Therefore, the ionization coefficient of the excited atoms is now derived using a treatment similar to that given in Ref. 10 for deriving s_i for a single step ionization process from the ground level. The equation for the ionization coefficient from an excited level is thus given by

$$s_{1*i} = 0.67 \times 10^6 (E_e)^{0.5}(k\beta_{*i}) \times (V_i - V_{1*} + 2E_e) \exp-(V_i - V_{1*})/E_e \tag{27}$$

where $(k\beta_{*i})$ is the ionization cross section from the excited states, in m²/v, of the impacting electron energy above the least energy required to ionize an excited atom, viz., $(V_i - V_*)$. This cross section is at least one to two orders of magnitude higher than the cross section for ionization from the ground state. (Von Engel,[5] e.g., gives the cross section for ionization of excited hydrogen atoms to be 16 times the cross section for ionization of hydrogen atoms in the ground state. For cesium the ratio is likely to be 100 or more.)

Taking $(k\beta_{*i}) = 10^{-17}$ m²/v, we get, for $\lambda_0 = 8521$ Å, $V_{1*} = 1.45$ v, $E_e = 0.118$ v, and $s_{1*i} = 0.646 \times 10^{-20}$ m³/sec. Similarly, for $\lambda_0 = 8943$ Å, $V_{2*} = 1.38$ v, $E_e = 0.118$ v, and we get $s_{2*i} = 0.367 \times 10^{-20}$ m³/sec. We now calculate the electron density resulting from ionization by electron-excited atom collisions, as given by Ref. 1:

$$n_e = (\Sigma n_* s_{i*}/\alpha_{6p}) = 2.44 \times 10^{15} \text{ electrons/m}^3$$

(using an $\alpha_{6p} = 0.588 \times 10^{-19}$ m³/sec). This value is of the same order of magnitude as the value of n_e which is derived from the test results (viz., $n_e = 4.27 \times 10^{15}$ electrons/m³).

Discussion

We have presented here the results that were obtained December 11, 1963, using the closed-loop magnetoplasmadynamic research facility described in Ref. 11. We have also presented a comparison of these results with the theory of nonequilibrium ionization considering collisional ionization processes from the ground state or from the resonance excited states.[1,5,10] These excited states exist as a result of the absorption by the cesium seed atom density of the spectral radiation, emitted by the magnetoplasmadynamic duct walls, over the broad resonance bandwidths (e.g., 50 Å[9]) of the two atomic cesium resonance lines at 8521 and 8943 Å.

Although the results were obtained at the low gas temperature of 863°K (approx 1100°F) or less (812°K or 1000°F) and the conductivities measured were only on the order of a mmho/m, nevertheless they represented one of the earliest experimental indications of the presence of nonequilibrium ionization conductivities under actual closed-loop magnetoplasmadynamic generator operating conditions, since the conductivities measured were more than two orders of magnitude higher than those that could possibly have been attained by thermal equilibrium ionization processes alone, at the gas temperature involved. As such, they are of significant scientific value, and their interpretation in the light of nonequilibrium ionization theories presents an invaluable guide for further high-temperature runs.

Higher temperature seeded runs were not possible at the time because of the main heater partial burnout, which took place in the preceding main heater trial runs. This burnout was probably due to the accidental dropping out, during transportation or installation of the assembly, of one of the ceramic blocks between the main heater leads which was intended to block the gas from bypassing its main flow passage through the heater elements. This caused a large fraction of the gas flow to bypass the heater. This in turn probably resulted in a temperature runaway in certain heater portions (#2 and #3) and a partial heater burnout. The block was restored to its original blocking position upon examination of the heater after the trial runs and prior to the test runs giving the results reported herein.

Complete burnout of the one remaining good heater (#1) during the runs reported here obviated the rise to higher gas temperatures. Results reported and discussed in this paper were obtained at temperatures of about 863°F (1100° F).

Because of the importance of the gas temperature in the interpretation and conclusions to be drawn from the test results, its value, as measured by the thermocouples imbedded in the duct insulation, was checked by comparing the heat input to the cooling water in the duct cover box main and after coolers with the difference between the enthalpy of the gas in the duct, based on the measured ga temperature, and the enthalpy of the gas at the after-cooler exit, based on the gas temperature as it left the after cooler. These evaluations verified the gas temperature measured within about ±30°K. Nevertheless, the thermal equilibrium conductivity at a gas temperature of 928°K (i.e., 65°K above the maximum measured value of 863°K) was evaluated using Saha's equation and found to be only 9.4 μmho/meter, as compared to the measured conductivity of 2.47 mmho/m, i.e., only 1/263 of the value evaluated from direct measurement of the currents by a microammeter.

Furthermore, if only thermal equilibrium conductivity existed, then the conductivity should be substantially uniform throughout the duct, and consequently the current densities measured in the x direction and y direction under virtual external short-circuit conditions in the Faraday and Hall connections should be related in accordance with Eqs. (1) and (2), when both ϵ_x and ϵ_y vanish, viz., by the relation $J_x/(-J_y) = \beta = \mu_e B = 4.44 \times 1.05 = 4.65$, at $B = 1.05$ weber/m^2 and $E_g = 0.0745$ v (863°K).

This, however, was not the case, and the current density ratio was 0.268 instead, because nonequilibrium ionization conductivity did not build up at the first electrode pair to its value at the seventh electrode pair, and the nonuniform conductivity along the duct caused a virtual load Hall field to exist with respect to the seventh electrode pair, hence, the low measured value of $(J_x/-J_y)$. This fact tends to refute any argument that the effect measured could possibly have been due to thermal equilibrium ionization alone.

One may conclude, from the comparison between measurements and theory, that the likely mechanism of ionization that took place was predominantly a collisional ionization mechanism from the resonance excited states of the two atomic cesium resonance lines at 8521 and 8943 Å ($V_* = 1.45$ and 1.38 volts, respectively). Although the number density of these excited states depends on the magnetoplasmadynamic duct wall temperature [see Eq. (26)], their ionization coefficient depends on the electron temperature [see Eq. (27)]. The electron temperature in turn depends on the gas temperature [see Eq. (21)] as well as on the factor β (which is the product of the electron mobility μ_e and the magnetic field B), the Mach number M, and the effective electrical load factors e_L and θ in the Faraday and Hall generator modes of operation, respectively [see Eqs. (5, 6, 9–11, and 21)]. Equation (21) indicates that the electron temperature will always be higher than the gas temperature. This is a necessary condition for the electrons to transmit the Joule heat of the generator to the gas atoms. But the amount that E_e is higher than E_g depends on the value of the electron mobility; or the magnitude of the collision frequency ν_c, since $\mu_e = e/(m_e\nu_c)$; the magnetic field B; the Mach number M; the effective Hall generator load factor θ; and the magnitude of the effective Faraday generator load factor e_L. In applying Eq. (21), one should not lose sight of the interrelation among these quantities and their effect on the power density and efficiency. Thus, a low load factor e_L would mean high electron temperature but also a low efficiency. A high Mach number M would mean a high ratio of E_e/E_g but would also mean a low value of E_g if the stagnation temperature T_s is limited by the long time endurance capability of the heat source materials, since $T_s = T_g[1 + (\gamma - 1)M^2/2]$. An effective loading in the Hall generator mode, whether due to finite electrode segmentation ($0 < r < 1$), external Hall load [Eq. (9)], or end leakage [Eq. (10)], would lower the difference between E_e and E_g and could obviate a useful rise in the electron temperature. For example, under a true Hall short circuit load connection $\theta = 0$, the ratio of electron to gas temperature would be down to 1.15 when $e_L = 0$. The corresponding electron temperature would be 993°K which could not possibly cause the level of conductivities derived from the test results. The factor β plays an important role, and under ideal Hall generator open-circuit condition, viz., $\theta = 1$, the ratio of electron to gas temperature approaches $[1 + 0.49\beta^2M^2(1 - e_L)^2]$, which, for a gas with constant cross section like helium, and negligible effect of the seed atom or ion scattering on the mobility, would lead to

$$(E_e/E_g) = 0.5 + 0.5\{1 + [10^6BM(1 - e_L)/q_gn_g]^2(1134/T_g)\}^{0.5}$$

where q_g is in m^2, n_g is in m^{-3}, and T_g is in °K which shows the dependance of E_e/E_g on the product q_gn_g.

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